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## Longitudinal Oscillations in Bounded Magnetoplasmas

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Fine structure in absorption due to Buchsbaum–Hasegawa modes is observed over a wider range of magnetic fields than previously reported ($\omega_c/\omega = 0.5$–$0.985$). The basic theory is satisfactory only near the cyclotron harmonic.

Resonant fine structure near the electron cyclotron harmonics has been observed in previous measurements of emission, absorption, and reflection of extraordinary electromagnetic waves in nonuniform plasma columns.[1–3] The resonances are attributed to longitudinal plasma waves which are excited in the upper hybrid resonant layer and propagate for $\omega_c/\omega \gtrsim 1/n(n = 2, 3, \cdots)$ in the overdense plasma region $\omega_p^2(r) > \omega^2 - \omega_c^2$ ($\omega_p$ is the plasma frequency, $\omega_c$ is the cyclotron frequency for electrons) at right angles to the magnetic field. When the phase satisfies a quantization condition,[3] standing waves can be set up giving rise to closely spaced absorption lines on the cold plasma absorption background. In this note we report observations of these resonances in the entire range between the second harmonic of the cyclotron frequency and the cyclotron frequency itself, in an afterglow plasma in which electron density and temperature are independently measured.

The present experimental apparatus has been described in more detail in previous papers.[4,5] The optimum experimental conditions for observing the resonances are the following: the plasma is created by rf breakdown (21 MHz, 500W pulses, 300 $\mu$sec length, 40 msec repetition time) in argon at $15 \pm 5$ mTorr gas pressure. The plasma column (10 mm i.d.) is coaxial to a uniform static magnetic field and passes perpendicularly through the narrow sides of an S-band waveguide. The plasma density $n_e$ and temperature $T_e$ in the first 2 msec of the afterglow are accurately measured with a time-gated radiometer.[4] These parameters decay simultaneously from $n_e \simeq 10^{11}$ cm$^{-3}$ and $T_e \simeq 6000$°K at $t_e = 100$ $\mu$sec to $n_e \simeq 10^9$ cm$^{-3}$ and $T_e \simeq 3000$°K at $t_e = 2$ msec.

The absorption of a weak microwave signal (3000 MHz, $-70$ dBm) is observed versus afterglow time. For $\omega_c/\omega < 1$, a background of strong absorption[5] due to upper hybrid resonances exists whenever a local electron density in the column satisfies the resonance condition $\omega_p^2(r,t_a) = \omega^2 - \omega_c^2$. As the density decays with aftergolw time $t_a$, the radial position $r$ of the resonant layer shifts to the column center ($\omega_c$ = const). The observed end point of absorption at times $t_{a0}$ occurs when $\omega_p^2(0, t_{a0}) = \omega^2 - \omega_c^2$. Thus, by varying either $\omega$ or $\omega_c$ the decay of the peak density $\omega_{p0}(t_a)$ is found.

For $\omega_c/\omega \gtrsim 0.5$ a fine structure in absorption is observed which, after careful alignment of the column axis with the magnetic field, can be seen in the entire range $0.5 \lesssim \omega_c/\omega \lesssim 0.985$. The fine structure always occurs close to the absorption end, near A in Fig. 2 of Ref. 5, i.e., when the hybrid layer lies near the column center. Collisional damping probably prevents formation of standing waves over larger distances.

Taking the end point of the fine structure as the time when the maximum upper hybrid frequency equals the signal frequency, the locus of the resonance peaks has been plotted in an $\omega_c/\omega$ versus $\omega_{p0}^2/\omega^2$ diagram, thereby presenting a comparison and extension of previous results.[2] In order to obtain the experimental points in this diagram, the spacing of the resonance peaks in time is converted into a spacing in the parameter $\omega_{p0}^2/\omega^2$ by means of the peak density decay curve. The results are shown in Fig. 1.

Buchsbaum and Hasegawa first derived a theory predicting the location of the resonances in the above diagram. Although their analysis was subsequently

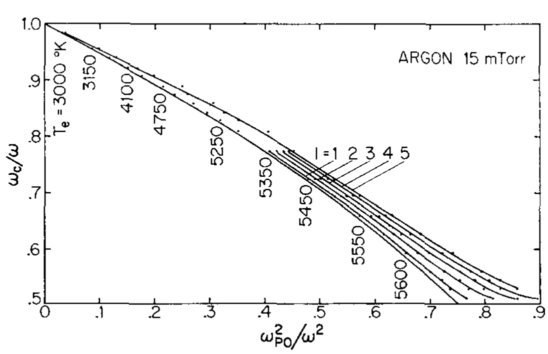


Fig. 1. Locus of the resonance peaks in the plane $\omega_c/\omega$ versus $\omega_{p0}^2/\omega^2$. The lower limit is given by the upper hybrid relation. The upper curve corresponds to the fifth resonance peak. For purposes of display the location of the lower order resonance peaks is only shown in the right half of the diagram.

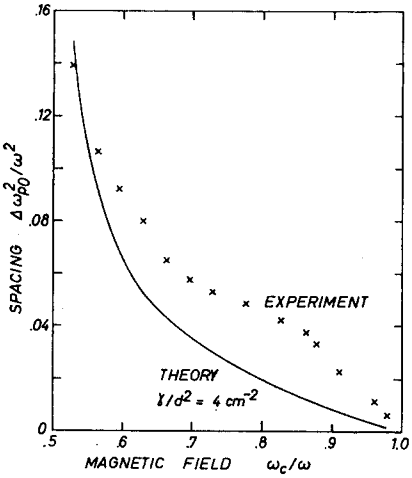


FIG. 2. Spacing in $\omega_{p0}^2/\omega^2$ between the fifth resonance peak and the upper hybrid resonance limit. Experimental data are taken from Fig. 1; the theoretical values are calculated from Buchsbaum and Hasegawa's theory, using measured, time-dependent electron temperature and a parabolic density profile ($\gamma = 1$).

refined,[6] the changes in the eigenvalues are small in most experimental cases and the basic theory may be considered as adequate. Their eigenvalue equation[2] may be written

$$\left(\frac{4\omega_c^2}{\omega^2} - 1\right)\left(\frac{\omega_c^2}{\omega^2} + \frac{\omega_{p0}^2}{\omega^2} - 1\right)\left(\frac{\omega_{p0}^2/\omega^2}{1 - \omega_c^2/\omega^2} - 1\right) = 12(l+1)^2\gamma\,\frac{kT_e}{m\omega^2 d^2}\,\frac{\omega_{p0}^2}{\omega^2},$$

where $kT_e$ is the electron energy, $m$ is the electron mass, $d$ is the column diameter, $l$ is the mode number for standing wave resonances, and $\gamma$ is a curvature parameter of the density profile. The latter is taken in the form $g(r) = (1 - \gamma r^2/d^2)^{-1} \simeq 1 - \gamma r^2/d^2$ when $r/d \ll 1$.

For purposes of comparison with the experiment we take the fifth resonant peak ($l = 5$) and calculate the spacing

$$\Delta\left(\frac{\omega_{p0}^2}{\omega^2}\right) = \left(\frac{\omega_{p0}^2}{\omega^2}\right)_{l=5} - \left(1 - \frac{\omega_c^2}{\omega^2}\right) \simeq \left(1 - \frac{\omega_c^2}{\omega^2}\right)\left(\frac{432\gamma(kT_e/m\omega^2 d^2)}{(4\omega_c^2/\omega^2 - 1)}\right)^{1/2}.$$

This spacing is proportional to the square root of the electron temperature and inversely proportional to the radius of curvature of the density profile in the column center.

Figure 2 shows the comparison between calculated and measured spacing for the fifth resonance line ($l = 5$, $d = 0.5$ cm, $\gamma = 1$, $\omega/2\pi = 3000$ MHz). Without adjusting the temperature and/or density profile as had to be done in previous experiments, good agreement is obtained in the vicinity of the second harmonic. However, as $\omega_c/\omega \to 1$, the spacing disagrees up to a factor of 3. This discrepancy cannot be explained by the decay of the electron temperature and a change in the density profile which can be measured independently in this experiment[4,5] ($T_e$ shown in Fig. 1; $\gamma \simeq 1$, see Ref. 5) but apparently reflects the limitations of the basic model. The present wide range of experimental data allows a more meaningful comparison between theory and experiment than previously possible and indicates the necessity of using a refined theory except in the close vicinity of the cyclotron harmonic ($\omega_c/\omega \gtrsim 0.5$).

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## Electrostatic Instabilities in Multicomponent Plasmas

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A commonly employed heuristic method for evaluating phase velocities and growth rates of ion and electron plasma oscillations is generalized here to multicomponent plasmas. The new formulation exhibits the correct Galilean transformation properties.

On those numerous occasions in plasma physics when the phase velocity is obtainable in approximate form by other means, it is often considered appropriate[1,2] to avoid the full Vlasov treatment[3] and to determine the growth rate of a longitudinal electrostatic wave by invoking the formula[1]

$$P = -\pi^{1/2}\omega \sum_j \omega_j^2(\omega - kV_j)\left(\frac{m_j}{2\kappa T_j k^2}\right)^{3/2} \epsilon_0 |e|^2 \cdot \exp\left[-\left(\frac{m_j}{2\kappa T_j k^2}\right)(\omega - kV_j)^2\right] \quad (1)$$